# Magnitude–Frequency Distribution of Injection-Induced Seismicity: Gutenberg–Richter or Safer?

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## 1. Introduction

**1.1 Magnitude–Frequency Distributions (MFDs) are important in understanding and monitoring induced seismicity**

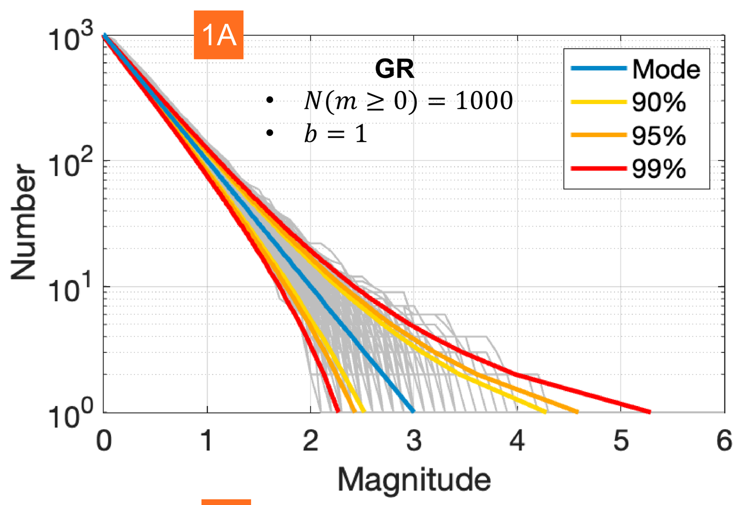
Significant progress has been made in the development of stress-based models to estimate the rate of induced earthquakes (e.g., Hager et al., 2021, Zhai et al., 2019, Kaveh et al., 2024, Acosta et al., 2023, Luu et al., 2022). However, these models are calibrated using small-magnitude earthquakes, and the use of MFD models is required to forecast earthquake rates at larger magnitudes. There are different types of MFDs. The most widely used are Gutenberg–Richter and tapered Gutenberg–Richter distributions.

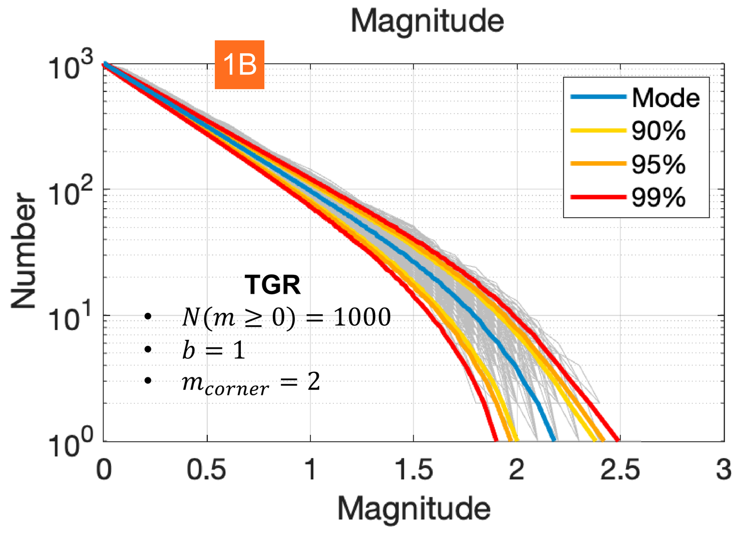
1) **Gutenberg–Richter (GR) distribution** (Gutenberg and Richter, 1944; Fig. 1A)

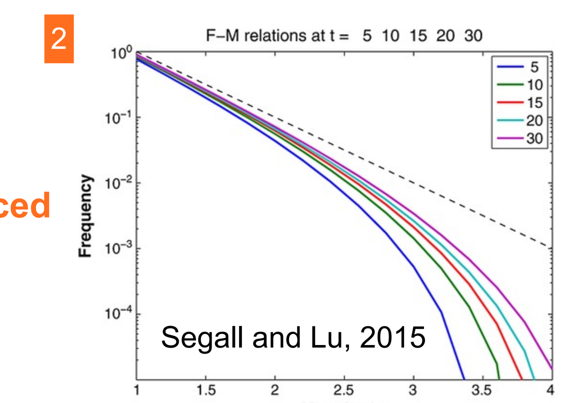
- Moment follows: $N(\geq M) = 10^{a-bm_C}\left(\frac{M_C}{M}\right)^{\beta}, \beta = \frac{2}{3}b, M \geq M_C$
- Predicted $m_{max}$ is $a/b$, $a$ is the total number of magnitudes $\geq 0$

2) **Tapered Gutenberg–Richter (TGR) distribution** (Kagan, 2002; Fig. 1B)

- Moment follows: $N(\geq M) = 10^{a-bm_C}\left(\frac{M_C}{M}\right)^{\beta} exp\left(\frac{M_C - M}{M_{corner}}\right), M \geq M_C$
- Predicted $m_{max}$ is smaller than $a/b$




**1.2 Different theories for the MFD and $m_{max}$ of induced seismicity**

**1) McGarr, 2014**
Upper bound of the maximum seismic moment is $M_0 = G\Delta V$.

**2) Shapiro et al., 2011; Shapiro et al., 2013**
Different MFDs may occur under different conditions for event triggering.

**3) Segall and Lu, 2015**
$m_{corner}$ increases with injection time (based on Shapiro et al., 2011) (Fig. 2).

**4) van der Elst et al., 2016**
$m_{max}$ of induced seismicity is as large as the GR distribution predicts.

**5) Galis et al., 2017**
The largest self-arrested rupture has a moment of $M_0^{arrest} = \gamma \Delta V^{3/2}$.

**6) Schultz, 2024**
Unbounded GR distribution outperforms the truncated GR distribution with volume-based $m_{max}$.

## 2. Statistical Methods

We have systematically explored the issues involved in differentiating between GR and TGR distributions, discussing the necessity, feasibility, and methodologies required. Some of the key findings are summarized as follows (Li and Avouac, BSSA, in press):

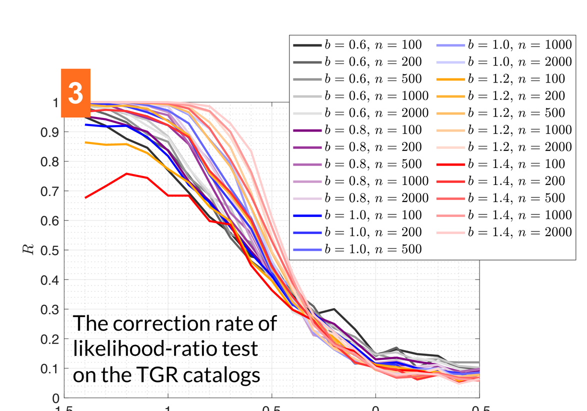
1) We derive semi-analytical confidence intervals for both distributions (Fig. 1), and show that insufficient sampling leads to an overestimation of large earthquakes. Thus, the apparent preference for the TGR distribution in the data cannot be attributed to inadequate sampling.
2) Likelihood-ratio test is the best method for distinguishing GR and TGR distributions.
3) The likelihood-ratio test seldom misclassifies GR catalogs as TGR catalogs if $m_C$ is determined correctly.
4) Using likelihood-ratio test, catalogs from TGR distribution can be misclassified as GR distribution if $m_{corner}$ is close to $a/b$ (Fig. 3).
5) We present an objective and automated framework (relying on random segmentation of the earthquake catalog by non-overlapping time windows) for characterizing the temporal evolution of MFDs (Fig.4).

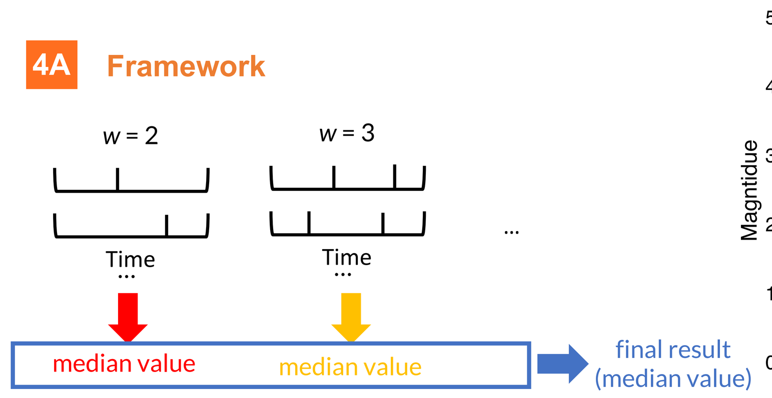
**Likelihood-Ratio Test (Kagan, 2002)**

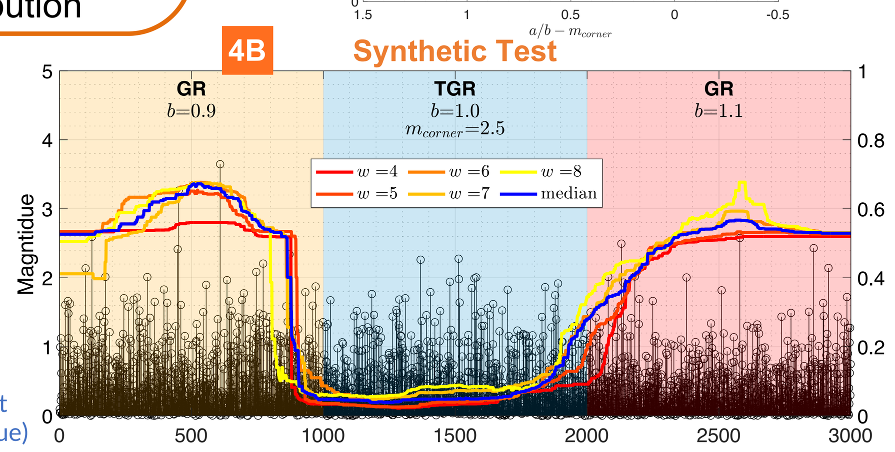
The probability $p$ the catalog belongs to GR distribution is determined by $2Y = \chi_p^2(1)$, where

$$Y = lnL_{TGR} - lnL_{GR}$$

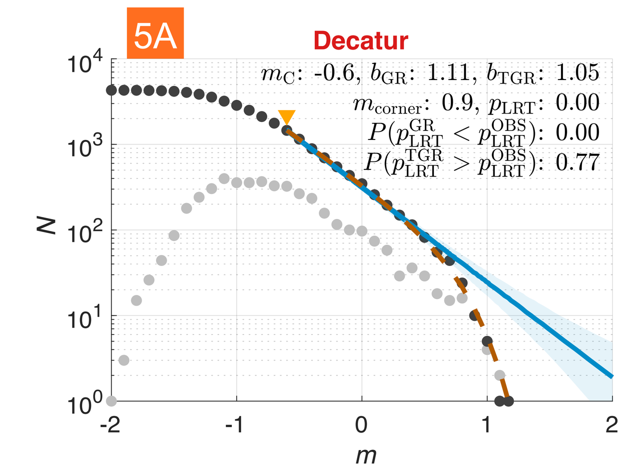
- $\chi_p^2(1)$: chi-squared distribution with one degree of freedom
- $lnL_{TGR}$, $lnL_{GR}$: log likelihood
  $p < \alpha$ (e.g., 0.05 or 0.1): TGR distribution

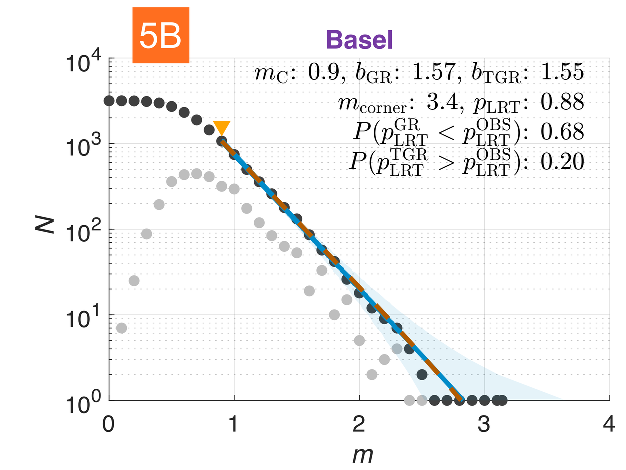


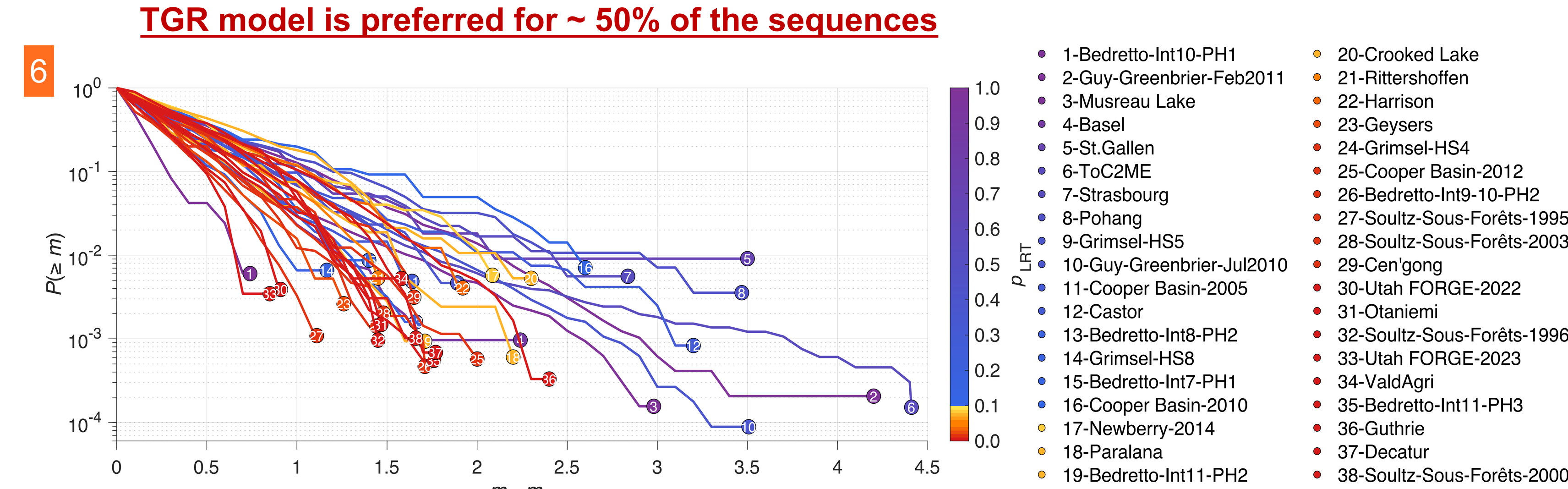




## 3. Global Observations

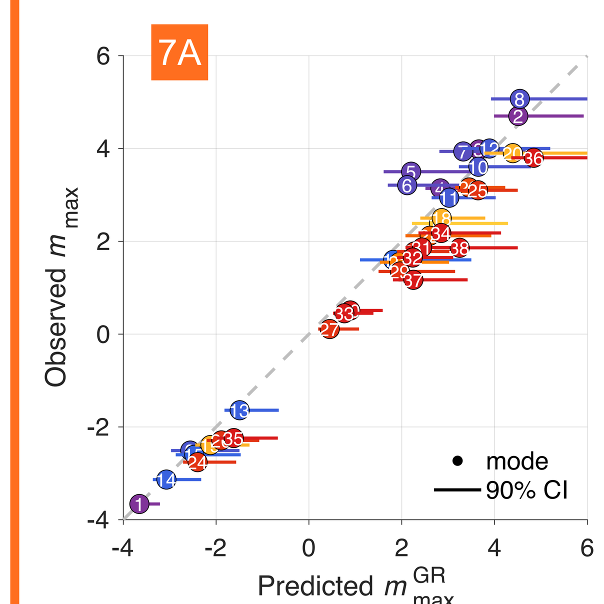


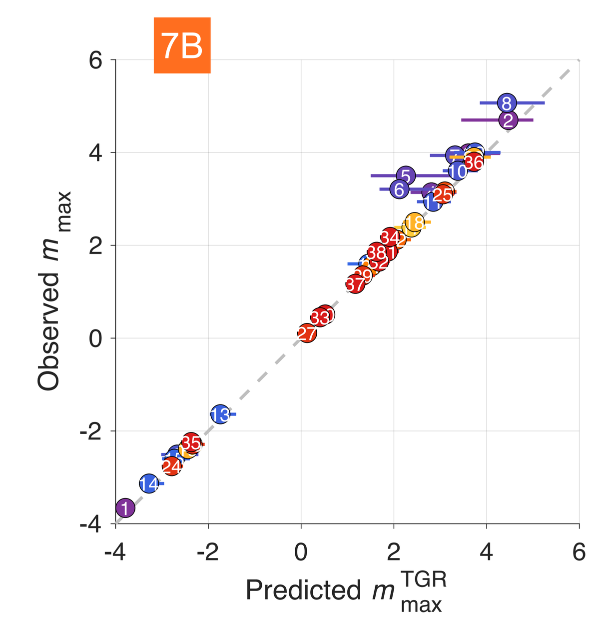




**TGR model is preferred for ~ 50% of the sequences**



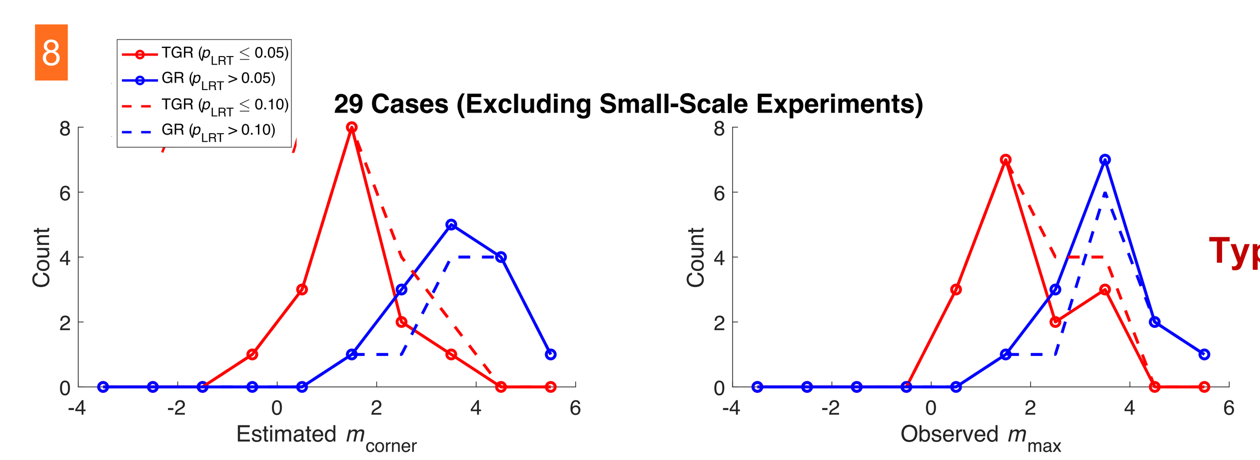

- $p_{LRT}$ = likelihood-ratio test $p$-value. red to yellow: TGR; blue to purple: GR



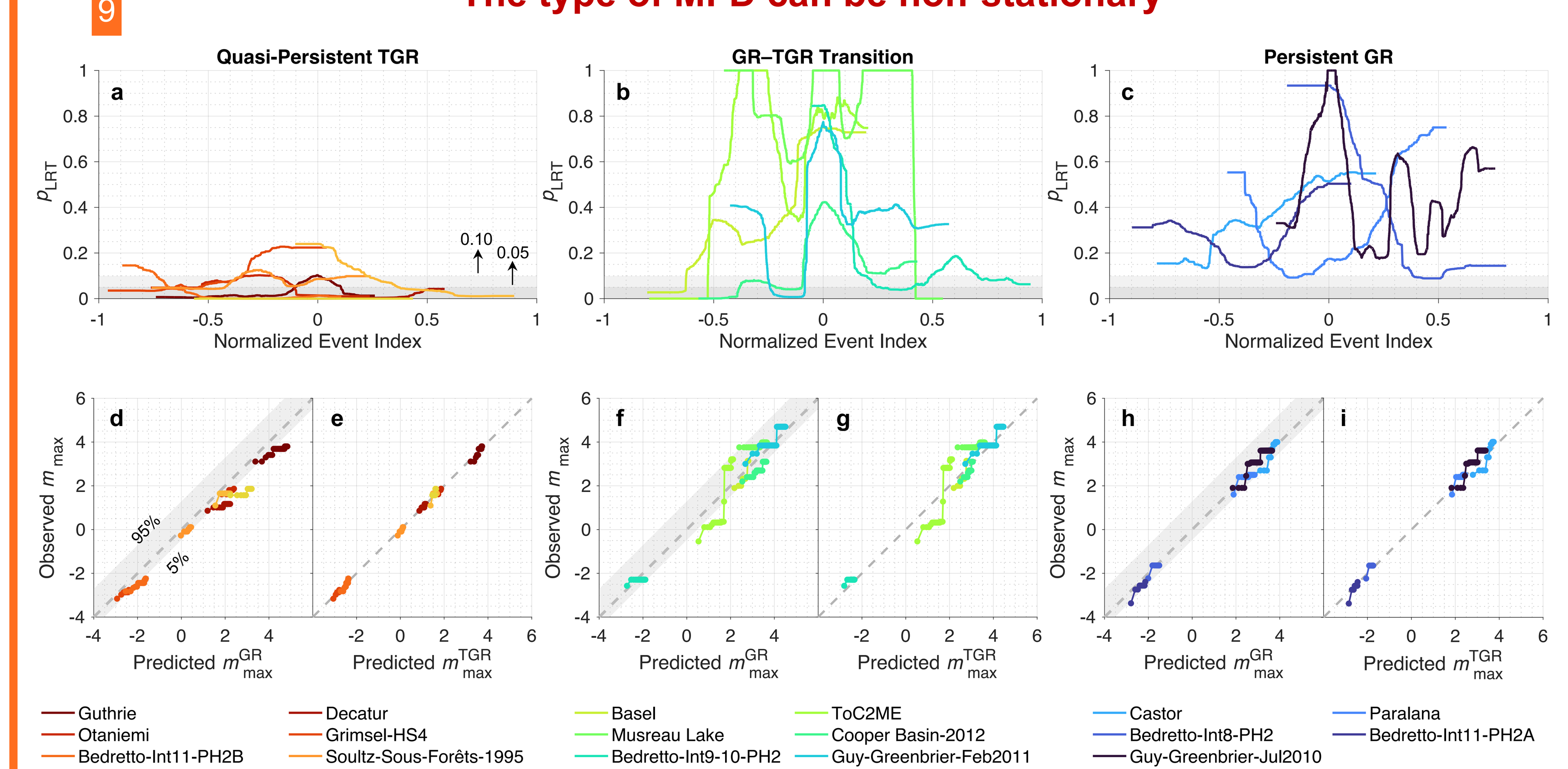

- **The hazard projections based on GR may be overstated.**
- **The TGR model explains why many injection-induced sequences remain below magnitude 2–3.**

Circle: predicted mode
Horizontal lines: 90% predicted confidence interval




**Typical Range for $m_{corner}$: 0–3**

**The type of MFD can be non-stationary**

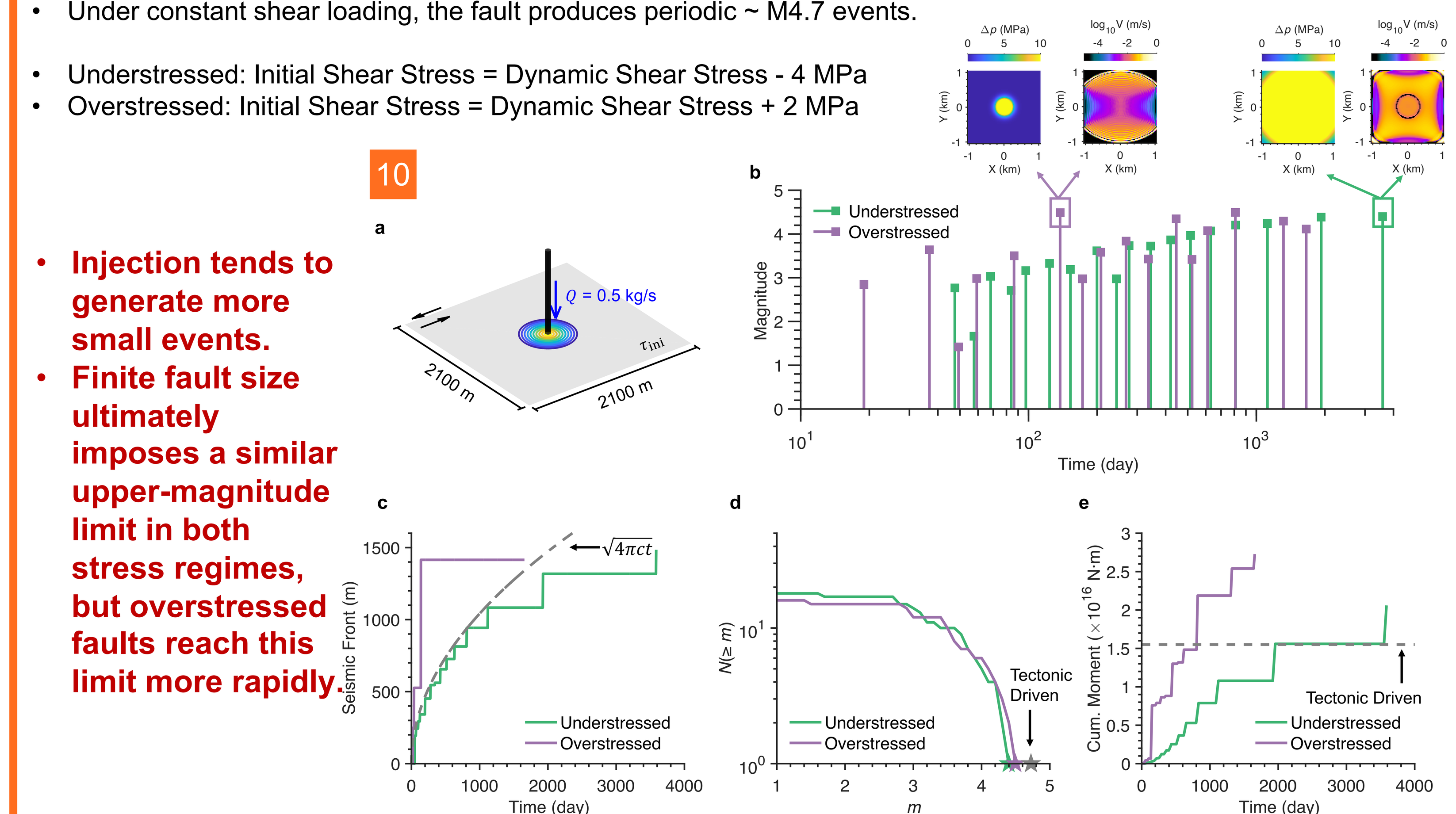



## 4. Physical Control

**Earthquake Simulator: Quake-DFN** (Im and Avouac, 2024)
2D rate-and-state fault; 3D elastic half-space; quasi-dynamic assumption

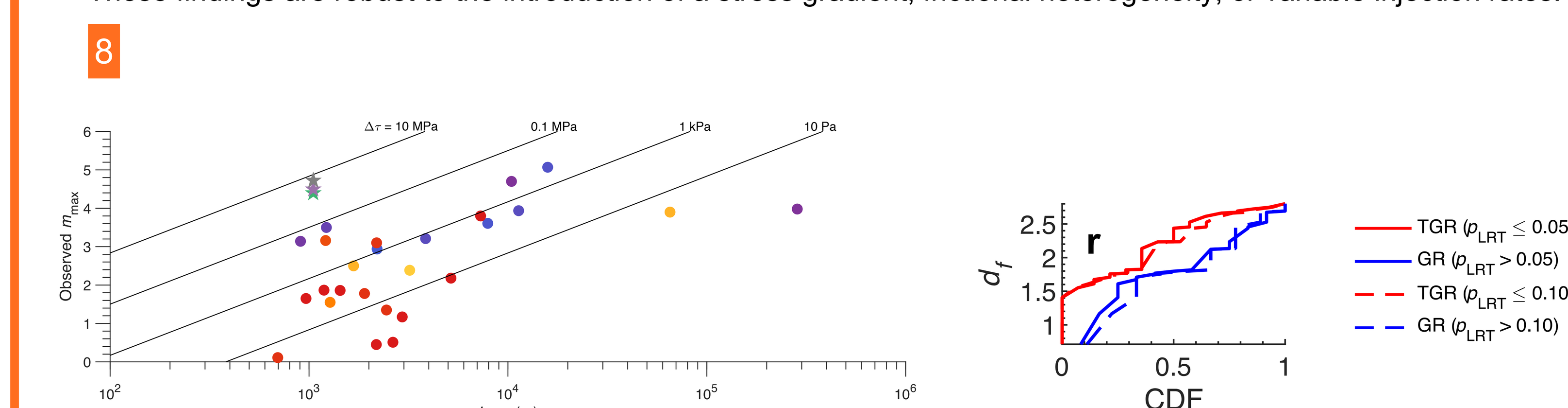
- Simulations continue until seismicity ceases, when the shear stress on the fault has entirely been released.
- Under constant shear loading, the fault produces periodic ~ M4.7 events.

- Understressed: Initial Shear Stress = Dynamic Shear Stress - 4 MPa
- Overstressed: Initial Shear Stress = Dynamic Shear Stress + 2 MPa




- **Injection tends to generate more small events.**
- **Finite fault size ultimately imposes a similar upper-magnitude limit in both stress regimes, but overstressed faults reach this limit more rapidly.**

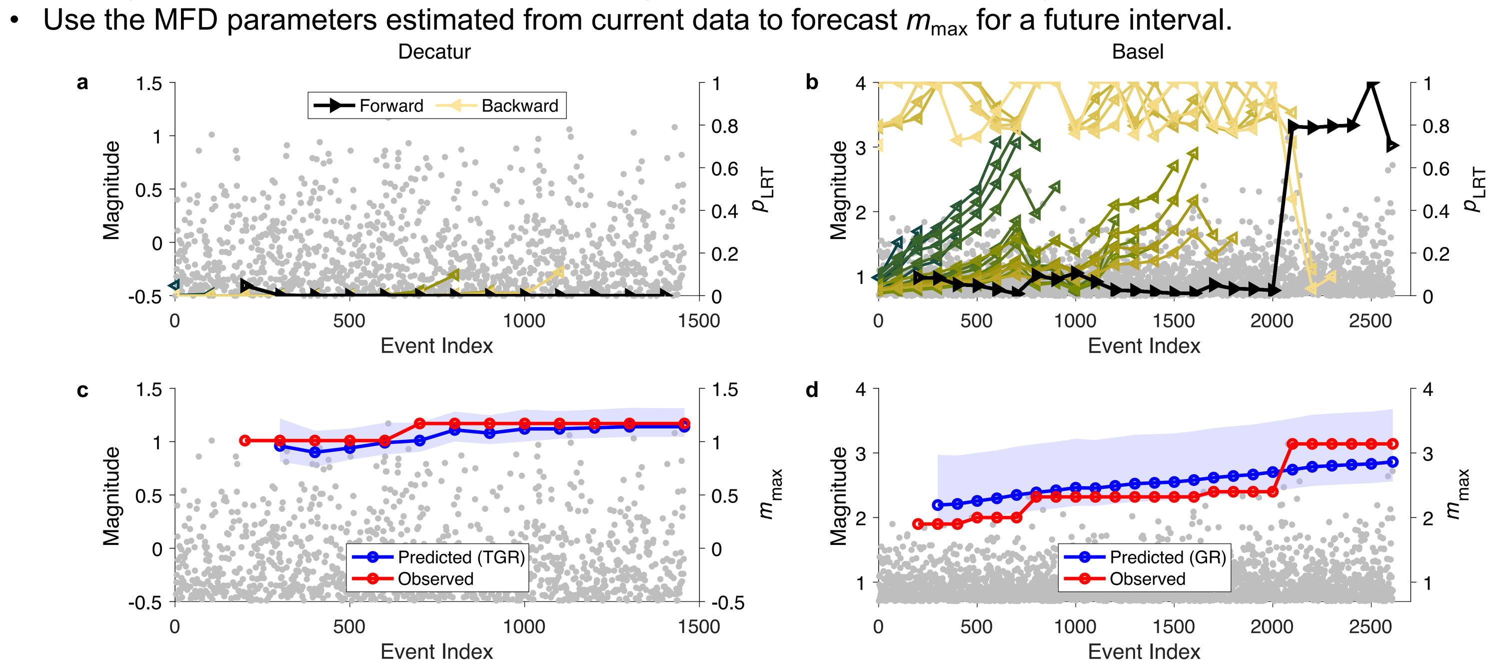
- These findings are robust to the introduction of a stress gradient, frictional heterogeneity, or variable injection rates.




- **In nature, the radius of maximum magnitudes is much smaller than the overall seismic cloud dimension.**
- **The largest magnitudes observed in nature are not controlled by the size of the stress perturbation.**
- **Instead, they are limited by both the injection tendency to generate smaller events and the distribution of seismogenic patches.**
- **This interpretation aligns with the higher fractal dimensions seen in TGR cases.**

## 5. Real-Time Monitoring

- Employ both forward and backward expanding windows to obtain the MFD type.
- Use the MFD parameters estimated from current data to forecast $m_{max}$ for a future interval.




## 6. Key Points

- About half of the induced earthquake sequences we analyzed follow TGR distributions.
- The prevalence of TGR behavior likely reflects both fluid-injection physics, which promotes small-event nucleation, and structural controls that limit fault activation.
- We propose a real-time method that integrates MFDs derived from observed seismicity with earthquake rate forecasts to predict future maximum magnitudes.

**Acknowledgements**

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